Electrical resistivity of thin wires at low temperatures: strained whiskers of copper

1991 J. Phys.: Condens. Matter 3 6403

(http://iopscience.iop.org/0953-8984/3/33/017)

J. Phys.: Condens. Matter 3 (1991) 6403–6413. Printed in the UK

Electrical resistivity of thin wires at low temperatures: strained whiskers of copper

D Movshovitz and N Wiser
Department of Physics, Bar-Ilan University, Ramat-Gan, Israel

Received 11 March 1991

Abstract. We explain quantitatively the data of Thummes and Kotzler for the temperature-dependent part of the electrical resistivity $\rho(T)$ for thin copper whiskers at low temperatures. The calculation includes the contribution to $\rho(T)$ due to non-resistive normal electron–electron scattering. In addition, we analyse the effect of straining the copper whiskers, showing why for one whisker, $\rho(T)$ *increased* upon straining, whereas for the other whisker, $\rho(T)$ *decreased* upon straining.

1. Introduction

Recent studies of the temperature-dependent part $\rho(T)$ of the electrical resistivity of the non-transition metals have emphasized two subjects:

(i) the contribution to $\rho(T)$ due to electron–electron scattering (for a review, see Kaveh and Wiser 1984), and

(ii) the significant difference between $\rho_{\text{expt}}(T)$ for bulk samples and for thin wires and films, for which electron–surface scattering is important (for a list of recent references, see Movshovitz and Wiser 1990a).

We here combine these two subjects in a study of the electron–electron scattering contribution to $\rho(T)$ at low temperatures for thin wires, with particular attention being paid to the role of normal electron–electron scattering. In addition, the analysis includes the large effect on $\rho(T)$ of straining the thin wires.

The motivation for the present study comes from some recent resistivity data of Thummes and Kotzler (1985), who measured $\rho(T)$ at very low temperatures for two copper whiskers. A particularly interesting aspect of this experiment is that after measuring $\rho(T)$, Thummes and Kotzler strained their copper whiskers and again measured $\rho(T)$. They found that for one of the whiskers, straining *increased* $\rho(T)$, whereas for the other whisker, straining *decreased* $\rho(T)$. Our analysis accounts quantitatively for these 'anomalous' results.

In section 2, we present the low-temperature resistivity data for the measured copper whiskers. These data are compared with our theory in section 3. The important contribution to $\rho(T)$ due to normal electron–electron scattering is reviewed in section 4. In section 5, the enhancement factors are discussed. The calculated results for $\rho(T)$ are presented for the unstrained copper whiskers in section 6, and for the strained whiskers in section 7. The angular dependence of the specularity parameter is discussed

0953-8984/91/336403 + 11 $03.50 © 1991 IOP Publishing Ltd

Table 1. Sample characteristics of the two copper whiskers (samples Cu 1 and Cu 2) measured by Thummes and Kotzler (1985), and for thick wires of Cu measured by Steenwyk *et al* (1981). For each wire, we list the diameter d, the residual resistivity ρ_0, and the measured coefficient $A_{expt} = \rho_{expt}(T)/T^2$. The last column lists our calculated values, A_{calc}, for comparison with the data.

Sample (surface)	Diameter (μm)	Annealed or strained	ρ_0 (nΩ cm)	A_{expt} (fΩ cm K^{-2})	A_{calc} (fΩ cm K^{-2})
Cu 1	22	Annealed	3.60	49 ± 3	44
(etched)		Strained	4.75	59 ± 2	58
Cu 2	6.9	Annealed	7.84	179 ± 2	172
(not etched)		Strained	8.45	82 ± 2	93
Cu 6 and Cu 11	~1500	Annealed	0.4	27 ± 1	—
(bulk samples)					

in section 8. In section 9, we describe the method of electron dynamics that was used to calculate $\rho(T)$. The summary follows in section 10.

2. Low-temperature measurements of $\rho(T)$ for copper whiskers

We begin by reviewing briefly the low temperature behaviour of $\rho(T)$ for bulk samples of non-transition metals. At sufficiently low temperatures ($T < 1.5$ K for Cu), the electron–phonon scattering contribution to $\rho(T)$ is negligible, leaving only the electron–electron scattering term, which exhibits the well-known quadratic temperature dependence,

$$\rho(T) = AT^2. \tag{1}$$

For copper, the experimental value of the coefficient is (Steenwyk *et al* 1981)

$$A_{expt} \simeq 27\,\text{f}\Omega\,\text{cm}\,\text{K}^{-2}. \tag{2}$$

Thummes and Kotzler (1985) measured $\rho(T)$ at low temperatures for thin copper whiskers. Below about 1.5 K, they found that $\rho(T)$ becomes quadratic in temperature, in accordance with (1). However, their value for A_{expt} for the whiskers (see table 1) was considerably larger than the bulk value—in fact 6 times larger for the thinner whisker (diameter $d = 6.9\ \mu$m). Such an enormous enhancement of A_{expt} was certainly unexpected.

To investigate this phenomenon further, Thummes and Kotzler strained the whiskers and remeasured A_{expt}. This important measurement again yielded unexpected results. As recorded in table 1, it was found that for the thicker whisker, A_{expt} *increased* upon straining, whereas for the thinner whisker, A_{expt} *decreased* upon straining.

The most puzzling aspect of these latter results is, of course, the fact that straining had the opposite effect on A_{expt} for the two whiskers. However, another anomalous feature relates to the large magnitude of the effect. Although the increase in the residual resistivity ρ_0 upon straining the thinner whisker was less than 10% (see table 1), the value of A_{expt} decreased to less than half its unstrained value.

3. Calculated results

3.1. Introduction

The data of Thummes and Kotzler (1985) can be explained quantitatively by including in the calculation of $\rho(T)$ for thin wires, the important contribution due to normal electron–electron scattering (NEES). In the last column of table 1, we list our calculated values A_{calc} for the two thin whiskers, both in the annealed state and in the strained state. The overall agreement between theory and experiment is evident from the table.

It is useful to begin with a qualitative explanation of the calculated results, leaving the quantitative details to be presented in subsequent sections.

3.2. The annealed copper whiskers

For the thinner whisker ($d = 6.9\ \mu\text{m}$) in the annealed state, the NEES term (which is non-resistive for bulk samples) dominates $\rho(T)$ and is primarily responsible for the very larger, factor-of-6 increase in A_{calc} over the bulk value. By contrast, for the thicker whisker ($d = 22\ \mu\text{m}$), the resistive umklapp electron–electron scattering (UEES) term dominates $\rho(T)$. As a result, the enhancement of A_{calc} over its bulk value is relatively modest—less than a factor of 2.

3.3. The strained copper whiskers

The effect of straining the whiskers is twofold. First, the contribution from non-resistive (in the bulk) scattering to $\rho(T)$ is sharply reduced. Second, the resistive scattering contribution to $\rho(T)$ is enhanced. These results (to be justified presently) are sufficient to explain the data.

For the thinner whisker, the first effect mentioned above is most important. The large non-resistive NEES contribution to $\rho(T)$ in the annealed state is largely suppressed by straining, and hence A_{calc} is significantly reduced—by more than a factor of 2—in agreement with experiment.

For the thicker whisker, the second effect mentioned above is most important. The dominant resistive scattering contribution to $\rho(T)$ is enhanced by straining, leading to an increase in A_{calc}—in agreement with experiment.

4. Normal electron–electron scattering

The NEES contribution to $\rho(T)$ for thin wires has recently been the subject of several investigations (Kaveh and Wiser 1984, De Gennaro and Rettori 1984, 1985, Wiser 1988, Gurzhi *et al* 1989a, b, Movshovitz and Wiser 1990a, b). Therefore, we need only summarize the main ideas.

For a thick wire, NEES makes no contribution to $\rho(T)$ because the total electron momentum is conserved at each NEES collision. However, for a thin wire, NEES *does* contribute to $\rho(T)$ by altering the direction of the electron trajectory. The physical

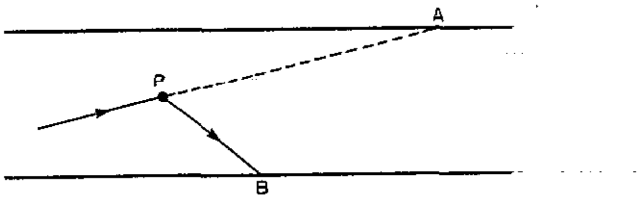

Figure 1. An electron undergoes a non-resistive NEES collision at point P, which causes the electron to strike the surface of the thin wire at point B, rather than at point A.

principle is illustrated in figure 1. An electron undergoes a non-resistive NEES collision at point P. This collision alters the direction of its trajectory, causing the electron to strike the surface of the wire at point B, rather than at the more distant point A. The electron mean free path is thereby shortened (PB < PA) and hence $\rho(T)$ is *increased*. Alternatively, the point A could be nearer to point P than is point B (PB > PA). In the latter case, the electron mean free path would be lengthened and $\rho(T)$ would be *decreased*. Thus, the net change in $\rho(T)$ due to NEES, denoted $\rho_{NEES}(T)$, can be either positive or negative, as is determined by averaging over all possible electron trajectories (Movshovitz and Wiser 1990b).

For the copper whiskers under consideration here, it is easy to show with the help of figure 1 that the sign of $\rho_{NEES}(T)$ will be positive.

The first point is that the electrical resistivity of a pure thin wire is due primarily to electron–surface scattering. Therefore, the dominant contribution to the current comes from those electrons whose trajectory makes a small angle with the wire axis because they can travel relatively long distances before colliding with the wire surface. Figure 1 depicts such a trajectory.

The second point is that NEES is a large-angle scattering process. Figure 1 depicts a scattering angle at point P of 45°, typical of NEES. As is clear from the figure, such a NEES collision shortens the electron mean free path, implying a positive contribution to $\rho_{NEES}(T)$. It is also clear from the figure that if the scattering angle were larger, or 'upwards' rather than 'downwards' (as shown), then the mean free path would be even shorter.

In summary, for thin whiskers, the important electron trajectories are always shortened by NEES, leading to a positive $\rho_{NEES}(T)$. Moreover, for thinner whiskers, the magnitude of $\rho_{NEES}(T)$ is greater.

Impurity scattering has an important effect on $\rho_{NEES}(T)$, because electron–impurity scattering causes the electron to lose all memory of its previous trajectory. Thus, the change in the electron mean free path due to non-resistive NEES, illustrated in figure 1, does not occur if an electron–impurity scattering event takes place anywhere along the trajectory. Indeed, this is the reason that $\rho_{NEES}(T)$ is appreciable only for thin wires, by which one means a wire whose diameter is comparable to or smaller than the mean free path λ_{imp} for electron–impurity scattering. For the copper whiskers under discussion here, the purity of the bulk material corresponds to $\lambda_{imp} = 240\ \mu m$ (Thummes *et al* 1985), which is very much larger than the diameter d of the whiskers.

The analysis shows that the value of $\rho_{NEES}(T)$ depends only on the ratio $\lambda_{imp}/d \equiv \lambda$. Therefore, one may write

Table 2. Calculated results for the two unstrained copper whiskers. The values of $\eta(\lambda)$ and $\gamma(\lambda)$ determine the values of $\rho_{UEES}(T)$ and $\rho_{NEES}(T)$ for each whisker, according to equations (3) and (6). The units for A_{calc} are fΩ cm K^{-2}.

Sample diameter	$\lambda = \lambda_{imp}/d$	$\eta(\lambda)$	$\gamma(\lambda)$	$\frac{\rho_{UEES} + \rho_{NEES}}{T^2} = A_{calc}$
Cu 1 ($d = 22\,\mu$m)	11	1.22	0.21	33 + 11 = 44
Cu 2 ($d = 6.9\,\mu$m)	35	2.79	1.79	75 + 97 = 172

$$\rho_{NEES}(T) = \gamma(\lambda) A_{NEES} T^2 \tag{3}$$

where $\gamma(0) = 0$ for bulk samples, and the quantity A_{NEES} simply denotes the electron mean free path for NEES, expressed in 'resistivity units',

$$A_{NEES} \equiv v_F m / ne^2 T^2 \lambda_{NEES}(T) \tag{4}$$

Kaveh and Wiser (1981) have shown that for copper,

$$A_{NEES} \simeq 2A_{UEES} = 54\ \mathrm{f}\Omega\ \mathrm{cm\ K}^{-2}. \tag{5}$$

This value for A_{NEES} has been confirmed by experiment (Kaveh and Wiser 1983).

5. Enhancement factors

For bulk samples of copper at very low temperatures, $\rho(T)$ arises solely from Umklapp electron–electron scattering, where $\rho_{UEES}(T)$ is given by equation (1) and the measured coefficient A_{expt} of equation (2) equals A_{UEES}. For thin wires and films, $\rho_{UEES}(T)$ is enhanced by electron–surface scattering (Sambles and Elsom 1980, Samples and Preist 1982, Thummes and Kotzler 1985, Movshovitz and Wiser 1990a). Thus, for thin wires, one may write

$$\rho_{UEES}(T) = \eta(\lambda) A_{UEES} T^2 \tag{6}$$

where the enhancement factor $\eta(\lambda)$ depends on the ratio $\lambda \equiv \lambda_{imp}/d$ for the particular wire under discussion. For bulk samples, $\lambda = 0$, $\eta(0) = 1$, and one recovers equation (1).

The complex theoretical task lies in the calculation of the λ-dependence of the enhancement factors $\gamma(\lambda)$ and $\eta(\lambda)$. In tables 2 and 3, we list our calculated values of $\gamma(\lambda)$ and $\eta(\lambda)$ for the unstrained (table 2) and the strained (table 3) copper whiskers measured by Thummes and Kotzler (1985). It is seen from the tables that in each case

$$\eta(\lambda) \simeq \gamma(\lambda) + 1. \tag{7}$$

This is not a numerical accident but can be shown to follow from a careful analysis of the calculation of the enhancement factors (Movshovitz and Wiser, to be published).

Table 3. Calculated results for the two strained copper whiskers. The values of $\eta(\lambda)$ and $\gamma(\lambda)$ determine the values of $\rho_{res}(T)$ and $\rho_{non\text{-}res}(T)$ for each strained whisker, according to equations (11) and (13). The units for A_{calc} are fΩ cm K^{-2}.

Sample (diameter)	$\lambda = \lambda_{bulk}/d$	$\eta(\lambda)$	$\gamma(\lambda)$	$\frac{\rho_{res} + \rho_{non\text{-}res}}{T^2} = A_{calc}$
Cu 1 ($d = 22\ \mu m$)	2.25	0.97	−0.05	59 − 1 = 58
Cu 2 ($d = 6.9\ \mu m$)	15	1.62	0.61	71 + 22 = 93

6. The unstrained copper whiskers

Our results for A_{calc} for the unstrained copper whiskers are listed in table 2. It is seen that for the thicker whisker, $\gamma(\lambda)$ is only 0.21. Therefore, $\rho_{UEES}(T)$ dominates $\rho(T)$ and the enhancement of A_{calc} over the bulk value is less than a factor of 2.

For the thinner whisker, $\gamma(\lambda)$ has the much larger value of 1.79. As a result, $\rho_{NEES}(T)$ is larger than $\rho_{UEES}(T)$, which is itself significantly enhanced by electron–surface scattering. The net effect of these two enhanced terms is to make A_{calc} more than six times larger than the bulk value.

From equations (5) and (7), it follows that $\rho_{NEES}(T)$ is larger than $\rho_{UEES}(T)$ whenever $\gamma(\lambda) > 1$. Therefore, for wires sufficiently thin and pure, λ satisfies this condition and the enhancement of A_{calc} over the bulk value will be very large.

7. The strained copper whiskers

7.1. Resistive electron scattering

When a bulk sample is strained, dislocations are generated which cause two effects. First, because dislocations are anisotropic scattering centres, the electron relaxation time is no longer isotropic over the Fermi surface. As a result, NEES *does* make a contribution to $\rho(T)$ even for a bulk sample and thus becomes partially resistive (res). The theory of this effect (Kaveh and Wiser 1980, 1982, 1984, Wiser 1984) leads to the following expression for $A_{res} \equiv \rho_{res}(T)/T^2$,

$$A_{res} = A_{UEES} + A_{NEES}\,(\rho_0/\rho_{dis})^{-2}. \tag{8}$$

In the denominator of the second term, ρ_{dis} denotes the electron–dislocation scattering contribution to the residual resistivity ρ_0 for the strained sample. For an annealed sample, $\rho_{dis} = 0$ and one recovers the usual result $A_{res} = A_{UEES}$. For the two strained copper whiskers, equation (8) yields $A_{res} = 60$ and 44 fΩ cm K^{-2}, respectively, whereas $A_{UEES} = 27$ fΩ cm K^{-2}.

Straining a wire leads to yet another effect. The increase in ρ_0 due to ρ_{dis} implies that the bulk electron mean free path is reduced from λ_{imp} to

$$\lambda_{bulk}^{-1} = \lambda_{imp}^{-1} + \lambda_{dis}^{-1}. \tag{9}$$

Therefore, the definition of λ must be correspondingly generalized to

$$\lambda \equiv \lambda_{bulk}/d. \tag{10}$$

For an unstrained whisker, equation (10) reduces to the previous result, $\lambda \equiv \lambda_{imp}/d$.

We may apply these results to thin wires by referring to equation (6). For strained whiskers, the quantity A_{UEES} is replaced by its generalization A_{res}, given by (8), yielding

$$\rho_{res}(T) = \eta(\lambda)A_{res}T^2 \tag{11}$$

where $\eta(\lambda)$ is the same function as before, but now the value of λ is obtained from equations (9) and (10), which includes the contribution to λ due to electron–dislocation scattering.

In table 3, we list for each strained whisker, the calculated values of λ and $\eta(\lambda)$, as well as the resulting values of $\rho_{res}(T)/T^2$.

7.2. *Non-resistive electron scattering*

For a strained sample, the non-resistive component of electron–electron scattering is no longer A_{NEES}. Because part of A_{NEES} has become resistive, the non-resistive (non-res) component is correspondingly reduced. Thus,

$$A_{non\text{-}res} = A_{NEES} - A_{NEES}(\rho_0/\rho_{dis})^{-2} \tag{12}$$

where the second term comes from equation (8). For an unstrained sample, $\rho_{dis} \simeq 0$ and $A_{non\text{-}res} = A_{NEES}$, as before.

For strained whiskers, equation (3) must be generalized to

$$\rho_{non\text{-}res}(T) = \gamma(\lambda)A_{non\text{-}res}T^2 \tag{13}$$

where $\gamma(\lambda)$ is the same function as before, with λ given by equations (9) and (10). For each strained whisker, the calculated values of $\gamma(\lambda)$ are listed in table 3, together with $\rho_{non\text{-}res}(T)/T^2$.

7.3. *Summary of effect of straining*

In summary, straining the whisker produces two effects. It increases A_{res}, which tends to increase A_{calc}, but it reduces λ and hence also reduces the enhancement factors $\gamma(\lambda)$ and $\eta(\lambda)$, which tends to decrease A_{calc}. For the thicker whisker, the former effect is dominant and hence A_{calc} increases. For the thinner whisker, the latter effect is dominant and hence A_{calc} decreases. This explains the data for the strained whiskers of copper.

8. Angle-dependent specularity parameter

In recent years, it has become clear (Sambles and Elsom 1980) that it is important to take account of the fact that electron–surface scattering is partially specular. In older work it was traditional to introduce a specularity parameter p, whose limiting values $p = 0$ and $p = 1$ denote respectively diffuse and specular electron–surface scattering. In practice, however, it was almost invariably assumed that $p \simeq 0$ is appropriate for resistivity calculations.

Soffer (1967) has pointed out that, in fact, p is not a constant, but rather that its value depends on the angle at which the electron strikes the wire surface. The Soffer expression for the angle-dependent specularity parameter is

$$p(\theta) = \exp[-(4\pi\alpha\cos\theta)^2] \tag{14}$$

where θ is the angle between the electron trajectory and the normal to the surface of the wire, and the surface-roughness parameter α is the ratio of the root mean square surface roughness to the electron de Broglie wavelength.

During the past decade, the pioneering work of Sambles and co-workers (Sambles and Elsom 1980, Sambles and Preist 1982, Sambles *et al* 1982) has shown that the angular dependence of the specularity parameter $p(\theta)$ must be included in the calculation of the resistivity for thin wires and films in order to obtain quantitatively reliable results.

Following these workers, we used equation (14) for $p(\theta)$ in the present calculation of $\rho_{\mathrm{NEES}}(T)$, taking α to be an adjustable parameter. One expects α to be of order unity (Sambles and Elsom 1980) for a wire whose surface has not been specifically roughened. We found that $\alpha = 2.2$ gave a good fit to the data for the thinner copper whisker. For the thicker whisker, the surface had been etched to roughen it, and therefore we used $\alpha = \infty$.

9. Calculation of $\rho_{\mathrm{NEES}}(T)$

9.1. *Introduction*

Our calculation of $\rho_{\mathrm{NEES}}(T)$ is based on an extension of the Chambers (1950) method of 'electron dynamics' to the case of non-resistive NEES. Since this method has been described in previous publications (Movshovitz and Wiser 1990a, 1990b), we need only summarize the principles involved.

To determine the change in the electron mean free path due to NEES, the electron mean free path is calculated with and without NEES, and then one takes the difference between these two quantities. We shall discuss each quantity in turn.

9.2. *Resisitive scattering*

Consider an electron with mean free path λ_{imp} for electron–impurity scattering. The probability $P(r)$ for the electron to travel a distance r without being scattered is

$$P(r) = \exp(-r/\lambda_{\mathrm{imp}}) \tag{15}$$

and the probability $\mathrm{d}Q$ for an electron–impurity collision to occur within the infinitesimal distance $\mathrm{d}r$ at a distance r from the origin of the electron trajectory is

$$\mathrm{d}Q(r) = (\mathrm{d}r/\lambda_{\mathrm{imp}}) \exp(-r/\lambda_{\mathrm{imp}}). \tag{16}$$

The average distance travelled by the electron before being scattered, Λ_0, is then given by

$$\Lambda_0 = \int r\, \mathrm{d}Q. \tag{17}$$

In a bulk sample, the limits on the integral are 0 and ∞. However, if the wire has a finite diameter, one obtains

$$\Lambda_0 = \lambda_{\mathrm{imp}}[1 - \exp(-R/\lambda_{\mathrm{imp}})] \tag{18}$$

where R is the distance to the wire surface in the direction of the electron trajectory.

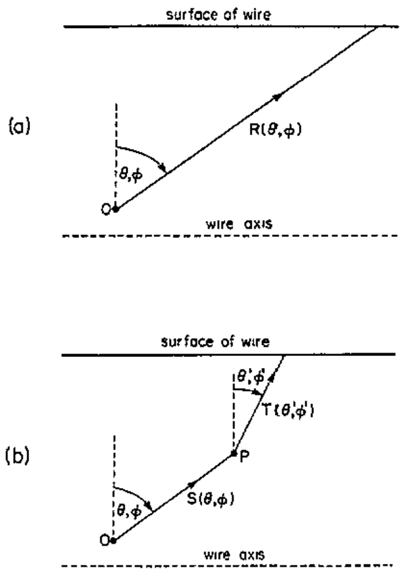

Figure 2. An electron trajectory in the direction θ, ϕ starting at the point 0. (a) Non-resistive scattering does not occur. The distance along this trajectory to the surface of the wire is $R(\theta, \phi)$. (b) Non-resistive scattering does occur. The electron travels a distance $S(\theta, \phi)$ and then undergoes a non-resistive collision at the point P, which alters the direction of its trajectory to θ', ϕ'. The distance along this new direction to the surface of the wire is $T(\theta', \phi')$.

The relevant geometry is illustrated in figure 2(a). The electron begins its journey at the point 0 and travels in the direction θ, ϕ, with $R(\theta, \phi)$ being the distance to the wire surface. To obtain the average mean free path for the electrons, $\Lambda_0(\theta, \phi)$ of (18) is integrated over all possible values of θ, ϕ and all possible origins 0 (Movshovitz and Wiser 1990a, 1990b).

The expression for Λ_0 given in (18) assumes that electron–surface scattering is totally diffusive. However, we take account of the reality that electron–surface scattering is, in fact, partially specular by introducing the specularity parameter p, which changes (18) into

$$\Lambda_0 = \lambda_{\rm imp}[1 - (1-p)\exp(-R/\lambda_{\rm imp})/(1 - p\exp(-R'/\lambda_{\rm imp}))] \qquad (19)$$

where R' is a distance related to R (for details, see Chambers 1950). It is readily seen from (19) that for diffuse scattering ($p = 0$), one recovers (18), whereas for specular scattering ($p = 1$), $\Lambda_0 = \lambda_{\rm imp}$ and electron–surface scattering has no effect at all.

An important feature of our calculation is that we take account of the angular dependence of the specularity parameter $p(\theta)$, whose value depends on the angle θ between the electron trajectory and the normal to the wire surface. The expression for $p(\theta)$ has been given in equation (14).

9.3. Non-resistive scattering

The presence of non-resistive NEES scattering events leads to two additional contributions to the electron mean free path, now denoted $\Lambda_{\rm NEES}$. The first contribution arises from the fact that the total electron mean free path without surface scattering, denoted $\lambda_{\rm tot}$, is shortened by NEES. Thus,

$$\lambda_{\rm tot}^{-1} = \lambda_{\rm imp}^{-1} + \lambda_{\rm NEES}^{-1} \qquad (20)$$

where $\lambda_{\rm NEES}$ is the electron mean free path due only to NEES. This implies that to obtain $\Lambda_{\rm NEES}$, one must replace $\lambda_{\rm imp}$ by $\lambda_{\rm tot}$ in the appropriate places in the calculation of Λ_0.

Since $\lambda_{\rm NEES} \gg \lambda_{\rm imp}$, one expands $\lambda_{\rm tot}$ to obtain

$$\lambda_{\rm tot} \simeq \lambda_{\rm imp} - \lambda_{\rm imp}^2/\lambda_{\rm NEES}. \tag{21}$$

The second term in $\lambda_{\rm tot}$ then leads to a contribution to $\Lambda_{\rm NEES}$ that is inversely proportional to $\lambda_{\rm NEES}$.

The second additional contribution to $\Lambda_{\rm NEES}$ arises from the type of electron trajectory that is illustrated in figure 2(b). As in figure 2(a) the electron begins its journey at the point 0, travelling in the direction θ, ϕ. However, after traversing a distance $S(\theta, \phi)$ and reaching the point P, the electron undergoes a NEES collision which alters the direction of its trajectory to θ', ϕ'. In this new direction, the distance to the surface of the wire is $T(\theta', \phi')$. Thus, the total distance from the initial point O to the wire surface is $S + T$, rather than R.

The type of trajectory shown in figure 2(b) will occur only if a NEES collision takes place at some point P, and hence its probability depends on the magnitude of $1/\lambda_{\rm NEES}$. Thus, we obtain a second contribution to $\Lambda_{\rm NEES}$ that is inversely proportional to $\lambda_{\rm NEES}$.

Figure 2(b) depicts the two segments of the electron trajectory, S and T, as if they were both lying in the same plane as the wire axis. This is, of course, not generally the case. Indeed, we found that it is very important to take into account the three-dimensional geometry of the scattering processes.

Having calculated $\Lambda_{\rm NEES}$, the change $\Delta\Lambda_{\rm NEES} = \Lambda_{\rm NEES} - \Lambda_0$ in the electron mean free path due to NEES can be expressed as a contribution to $\rho(T)$,

$$\rho_{\rm NEES}(T) = -C\Delta\Lambda_{\rm NEES}/\Lambda_0^2 \tag{22}$$

where $C \equiv mv_{\rm F}/ne^2$. The quantity Λ_0 appearing in (22) denotes the value obtained from $\Lambda_0(\theta, \phi)$ in (19) after integrating over all possible values of θ, ϕ; similar remarks apply to $\Lambda_{\rm NEES}$. Combining (22) with the result

$$\Delta\Lambda_{\rm NEES} \propto 1/\lambda_{\rm NEES} \tag{23}$$

yields the temperature dependence of $\rho_{\rm NEES}(T)$,

$$\rho_{\rm NEES}(T) \propto \lambda_{\rm NEES}^{-1}(T) \propto T^2. \tag{24}$$

The constant of proportionality in (24) has been denoted by $\gamma(\lambda)$ in equations (3) and (13), and has been calculated as a function of λ. The results for the measured copper whiskers, both annealed and strained, are listed in tables 2 and 3.

10. Summary

We have calculated the electron–electron scattering contribution to the low-temperature electrical resistivity of thin wires of copper. The new feature of our calculation is that we took explicit account of non-resistive normal electron–electron scattering. Applying our results to the recent low-temperature resistivity data for two copper whiskers, measured both in the annealed and in the strained states, yields quantitative agreement with experiment. In particular, the theory explains why, for one whisker, $\rho(T)$ increased upon straining whereas, for the other whisker, $\rho(T)$ decreased upon straining.

Acknowledgment

It is a pleasure for N W to acknowledge the hospitality of the Cavendish Laboratory of the University of Cambridge where this paper was written during his summer visit.

References

Chambers R G 1950 *Proc. R. Soc.* A **202** 378
De Gennaro S and Rettori A 1984 *J. Phys. F: Met. Phys.* **14** L237
—— 1985 *J. Phys. F: Met. Phys.* **15** 2177
Gurzhi R N, Kalinenko A N and Kopoliovich A I 1989a *Solid State Commun.* **72** 777
—— 1989b *Zh. Eksp. Teor. Fiz.* **96** 1522 (Engl. Transl. 1989 *Sov. Phys.–JETP* **69** 863)
Kaveh M and Wiser N 1980 *J. Phys. F: Met. Phys.* **10** L37
—— 1981 *J. Phys. F: Met. Phys.* **11** 1207
—— 1982 *J. Phys. F: Met. Phys.* **12** 935
—— 1983 *J. Phys. F: Met. Phys.* **13** L249
—— 1984 *Adv. Phys.* **33** 257
Movshovitz D and Wiser N 1990a *J. Phys.: Condens. Matter* **2** 8053
—— 1990b *Phys. Rev.* B **41** 10513
Sambles J R and Elsom K C 1980 *J. Phys. F: Met. Phys.* **10** 1487
Sambles J R and Preist T W 1982 *J. Phys. F: Met. Phys.* **12** 1971
Sambles J R, Elsom K C and Preist T W 1982 *J. Phys. F: Met. Phys.* **12** 1169
Soffer S B 1967 *J. Appl. Phys.* **38** 1710
Steenwyk S D, Rowlands J A and Schroeder P A 1981 *J. Phys. F: Met. Phys.* **11** 1623
Thummes G and Kotzler J 1985 *Phys. Rev.* B **31** 2535
Thummes G, Kuckhermann V and Mende H H 1985 *J. Phys. F: Met. Phys.* **15** L65
Wiser N 1984 *Contemp. Phys.* **25** 211
—— 1988 *Phys. Scr.* **37** 605